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Effect of a dynamic absorber on friction-induced vibration of a rectangular plate†

Hiroki Mori[1,*], Oleksandr Mikhyeyev[1], Takuo Nagamine[1], Mizue Mori[2] and Yuichi Sato[1]

[1]*Graduate School of Science and Engineering, Saitama University, 255, Shimo-Okubo, Sakura-ku, Saitama-shi, Saitama 338-8570, Japan*
[2]*Nissan Diesel Motor Co., Ltd., 1-1, Ageo-shi, Saitama 362-8523, Japan*

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Abstract

When a plate-like object is rubbed by rubber, friction-induced vibration is generated. To reduce the friction-induced vibration, we experimentally investigate the characteristics of the vibration of a rectangular glass plate. The results show that the frequency of the friction-induced vibration is almost the same as the natural frequency of a glass plate. The vibration is generated when the natural frequency of a rubbing system is close to that of a glass plate. We then examine the effect of a dynamic absorber mounted on the glass plate. The results demonstrate that the damping of a dynamic absorber is effective for suppressing the friction-induced vibration. Numerical simulation is also performed using a simplified analytical model. The calculated results agree qualitatively with the experimental ones.

Keywords: Self-excited vibration; Friction; Dynamic absorber; Sound pressure; Bouncing

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1. Introduction

Friction-induced vibration has been studied by many researchers. Adams [1] studied the dynamic characteristics of a flat body sliding on a rough surface. He showed the effects of friction coefficient, foundation stiffness, and asperity spring stiffness on stability. Ouyang and Mottershead [2] analyzed the instability of the transverse vibration of a circular disk under rotating two sliders. They indicated that the damping of the disk and the sliders is effective in reducing the unstable region. Nakano et al. [3] applied a dynamic absorber to prevent squeal in disk brakes. They showed that the damping of a dynamic absorber is not necessary for suppressing low frequency squeal in a car disk brake, whereas it is necessary for suppressing squeal in a bicycle disk brake.

When a plate-like object is rubbed by rubber, friction-induced vibration is generated, resulting in noise. An example is the noise induced by a closing window of an automobile. To reduce the friction-induced vibration, we experimentally investigate the characteristics of the vibration of a rectangular glass plate and examine the effect of a dynamic absorber mounted on the glass plate. Additionally, an analytical study is performed and compared with experimental results.

2. Experimental apparatus

Fig. 1 shows the experimental apparatus, which consists of a rectangular glass plate and a rubbing mechanism. The glass plate is suspended horizontally by four metal wires, which are inclined so that the glass plate moves only in one direction. The wires are located at nodal lines of the first vibration mode of the glass plate as shown in Fig. 2(a), since the first vibration mode is induced by rubbing. The dimensions and parameters of the glass plate are shown in Fig. 2(a) and Table 1. As shown in the table, the first natural frequency of the glass plate is 133 Hz. The rubbing mechanism consists of a rubber ball supported by a beam. The end of the beam is firmly fixed. When the glass plate moves horizontally, self-excited vibration occurs as well as noise. Figs. 2(a) and 2(b) show the positions of a dynamic absorber (DA), rubbing area, and sensors by which the acceleration of the glass plate and the sound pressure are measured. Fig. 2(c) shows a schematic of a dynamic absorber consisting of two metal plates, a rubber block, and a weight. Table 2 shows the parameters of the dynamic absorbers we used. Values in square brackets denote equivalent masses, which are calculated by Rayleigh's method using a static deflection curve of a cantilever beam with a concentrated mass at its end.

3. Results

Fig. 3 shows the waveforms of acceleration of a glass plate and sound pressure in the case without a dynamic absorber, as

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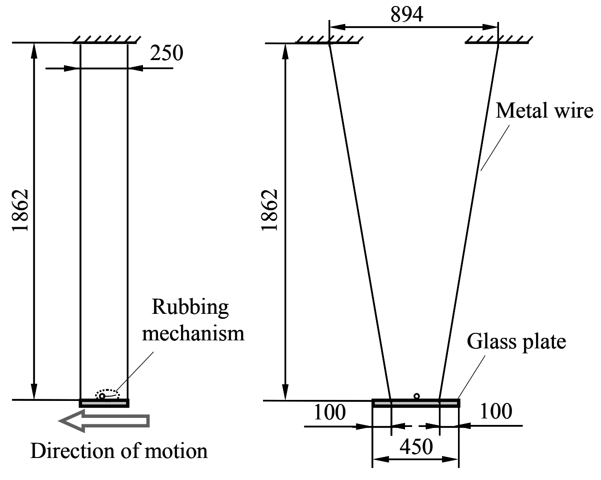
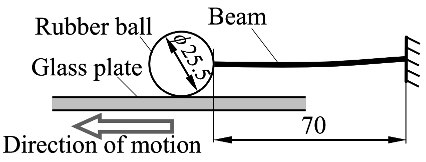
[*] Corresponding author. Tel.: +81 48 858 3730, Fax.: +81 48 858 3730
E-mail address: hmori@mech.saitama-u.ac.jp

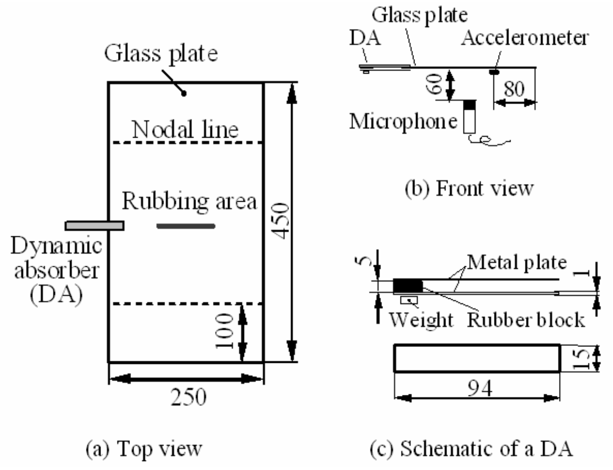


(a) Front view (b) Side view



(c) Rubbing mechanism

Fig. 1. Experimental apparatus.



(b) Front view

(a) Top view (c) Schematic of a DA

Fig. 2. Glass plate, sensors, and dynamic absorber (DA).

Table 1. Parameters and modal characteristics of the glass plate.

Length (mm)	450	Young's modulus (GPa)	71.6
Width (mm)	250	Poisson's ratio	0.23
Thickness (mm)	4.85	1st natural frequency (Hz)	133
Mass (kg)	1.365	Damping ratio	0.001

Table 2. Parameters of dynamic absorbers. (Values in square brackets denote calculated equivalent masses.)

No.	Mass (g)	Natural frequency (Hz)	Damping ratio
#1	27.0 [14]	139	0.06
#2	27.0 [14]	134	0.03
#3	25.5 [12]	131	0.02
#4	25.3 [12]	132	0.003
#5	18.7 [6]	132	0.001
#6	18.4 [5]	130	0.02

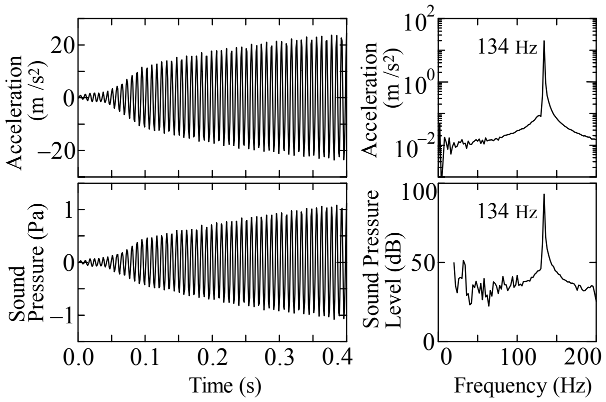


Fig. 3. Vibration of a glass plate without a dynamic absorber.

well as their frequency spectra. The background noise in the experimental room was less than 30 dB. The similarity between the two waveforms suggests that the sound is caused by the vibration of the glass plate. The vibration intensifies by rubbing, and its frequency is 134 Hz, which is almost the same as the first natural frequency of the glass plate. Through observation using a high-speed video camera, we confirmed that the rubber ball bounces on the glass plate, and the rotational motion of the ball is induced as well as the translational motion.

Fig. 4 shows the frequency and acceleration amplitude of the glass plate, the second natural frequency of the rubbing mechanism, and the first natural frequency of the glass plate. The beam length was changed from 30 mm to 100 mm. We used two types of the rubbing mechanisms with different masses, which are denoted as Types A and B in Fig. 4. The size of the circle represents the acceleration amplitude. The cross marks indicate that the acceleration amplitude is less than $5\,\mathrm{m/s^2}$, and an arrow indicates that the vibration of high frequency (≈ 340 Hz) is induced. As shown in the figure, the second natural frequency of the rubbing mechanism changes with the beam length. We note that a large vibration is induced when the second natural frequency of the rubbing mechanism is close to the first natural frequency of the glass plate in both cases of Types A and B.

Fig. 5. shows the acceleration amplitude of the glass plate with a dynamic absorber. The result of the case without a dynamic absorber is also shown in Fig. 5. We note that an absorber with high damping ratio reduces the vibration. The effect of the mass on the vibration is small compared with that of the damping ratio in the range of the experiment. The calculated result plotted in Fig. 5 is discussed in the next section.

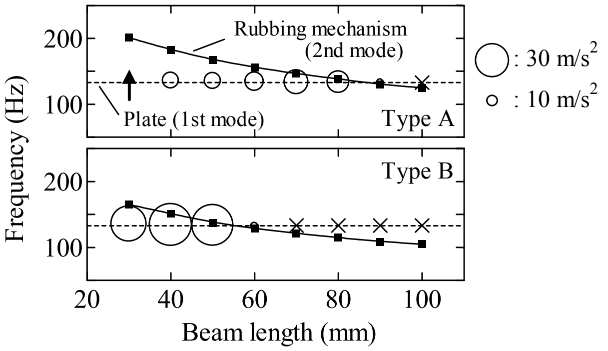


Fig. 4. Vibration amplitude and frequency.

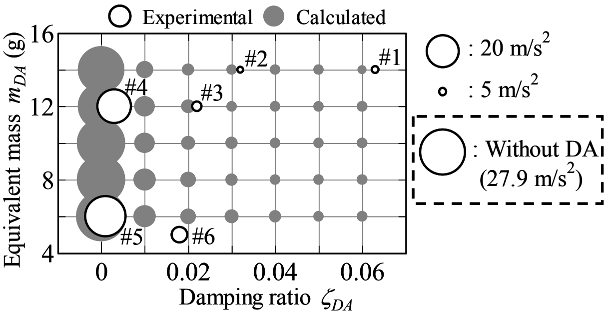


Fig. 5. Effect of modal mass and damping ratio.

4. Discussion

Finally, we consider an analytical model that explains the basic features of the experimental results. As shown in Fig. 6, the glass plate is assumed to be a single-degree-of-freedom system consisting of the modal mass m_0, modal stiffness k_0, and modal damping c_0. The rubber ball is modeled as a rigid sphere of mass m_1 and radius a supported by a flexible beam of length l and bending stiffness EI. By considering that the ball is bouncing, two models are used, which are switched depending on whether the ball is in contact with the plate or not, as shown in Figs. 6(a) and 6(b). For simplicity, the contact condition is modeled by the linear stiffness k_c and linear damping c_c. Further, we assume that a rubber ball in contact state does not slip on the glass plate. The velocity V of the glass plate is assumed to be constant. A dynamic absorber is modeled as a single-degree-of-freedom system, as shown in Fig. 6.

The equations of motion for the translational motion of the rubber ball, the glass plate, and the dynamic absorber are derived as follows:

$$m_1\ddot{y} + c_1\dot{y} + (12EI/l^3)y - (6EI/l^2)\theta - F_n = 0 \quad (1)$$

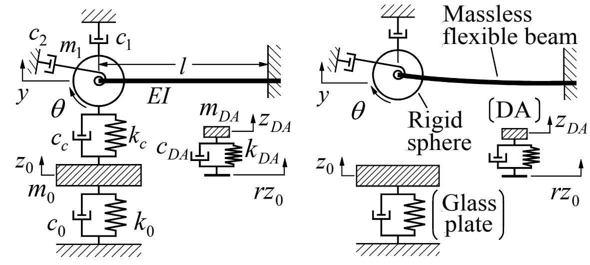
$$m_0\ddot{z}_0 + (c_0 + r^2 c_{DA})\dot{z}_0 - rc_{DA}\dot{z}_{DA} + (k_0 + r^2 k_{DA})z_0 - rk_{DA}z_{DA} + F_n = 0 \quad (2)$$

$$m_{DA}\ddot{z}_{DA} + c_{DA}(\dot{z}_{DA} - r\dot{z}_0) + k_{DA}(z_{DA} - rz_0) = 0 \quad (3)$$

where, F_n is the normal force at the contact point of the ball and the plate, and r is the displacement ratio determined from the mode shape of the glass plate. When the ball is in contact with the plate (Fig. 6(a)), the rotational motion of the ball and F_n are given by

Table 3. Parameters used for calculation.

a (mm)	12.8	ω_0 (rad/s)	133×2π	ζ_0	0.001
l (mm)	82.8	ω_b (rad/s)	13×2π	ζ_1	0.005
m_0 (kg)	1.06	d_s (mm)	0.023	ζ_2	0.01
m_1 (g)	8.1	V (m/s)	0.07	ζ_c	0.03
k_c (N/m)	1.7×10^4	r	1.2		



(a) Contact state

(b) Bouncing state

Fig. 6. Analytical models.

$$\dot{\theta} = V/a \quad (4)$$

$$F_n = -c_c(\dot{y} - \dot{z}_0) - k_c(y - z_0)\,. \quad (5)$$

When the ball is bouncing (Fig. 6(b)), they are given by

$$J\ddot{\theta} + c_2\dot{\theta} - (6EI/l^2)y + (4EI/l)\theta = 0 \quad (6)$$

$$F_n = -k_c d_s\,, \quad (7)$$

where d_s represents the static deflection in the equilibrium condition, and $J = (2/5)m_1a^2$.

The Runge-Kutta method is used for numerical simulation. The parameter values we used for simulation are shown in Table 3. Here, the modal mass m_0 of the glass plate and r are determined by FEM analysis using a mesh of 10×8 thin rectangular elements. The following parameters are introduced:

$$\zeta_0 = c_0/(2m_0\omega_0),\ \zeta_1 = c_1/(2m_1\omega_b),\ \zeta_2 = c_2/(2J\omega_b),$$
$$\zeta_c = c_c/(2m_1\omega_c), \omega_0 = \sqrt{k_0/m_0},\ \omega_b = \sqrt{3EI/m_1l^3},$$
$$\omega_c = \sqrt{k_c/m_1},\ "\cdot" = d/dt. \quad (8)$$

The calculated waveform of the glass plate without a dynamic absorber is shown in Fig. 7, which agrees with the experimental result shown in Fig. 3. The calculated amplitude of the glass plate with a dynamic absorber is plotted in Fig. 5. Here, the angular natural frequency of a dynamic absorber is set to $\omega_{DA} = 133\times 2\pi$ rad/s. The calculated result indicates that the damping of a dynamic absorber is effective for reduce-

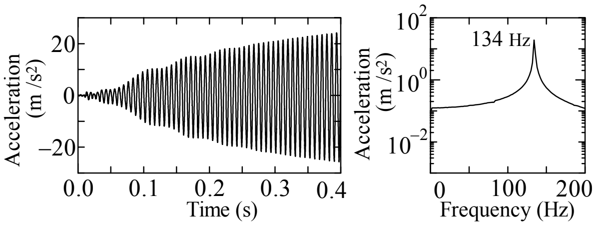


Fig. 7. Calculated acceleration of a glass plate.

ing the vibration. This agrees qualitatively with the experimental results.

5. Conclusions

To reduce the friction-induced vibration, the effect of a dynamic absorber mounted on a rectangular glass plate was investigated. From experimental study, we obtain the following conclusions:

(1) The damping of a dynamic absorber reduces the friction-induced vibration in the range of the experiment.
(2) The vibration is induced when the natural frequency of the rubbing mechanism is close to that of the glass plate.

In addition, we obtain an analytical model assuming that the rubber ball bounces on the glass plate. Calculated results agree qualitatively with the experimental ones.

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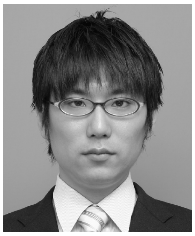

Hiroki Mori received his D.E. degree in Mechanical Engineering from Kyushu University, Japan, in 2006. He is currently an Assistant Professor at Saitama University.

Oleksandr Mikhyeyev received his Ph.D. degree in Mechanical Engineering from Saitama University, Japan, in 2009.

Takuo Nagamine received his D.E. degree in Mechanical Engineering from Saitama University, Japan, in 2005. He is currently an Associate Professor at Saitama University.

Mizue Mori received her M.E. degree in Mechanical Engineering from Kyushu University, Japan, in 2006. She is currently an Engineer at Nissan Diesel Motor Co., Ltd.

Yuichi Sato received his D.E. degree in Mechanical Engineering from Tokyo Institute of Technology, Japan, in 1978. He is currently a Professor at Saitama University.